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SPACE ELEVATOR TETHER ATMOSPHERIC WIND LOADING AND A CABLE LIFT CONCEPT

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Abstract

The present concept for an Earth Space Elevator system includes a ribbon tether of the order of 1 metre wide. This width is considered adequate for debris impact survival and to enable ascent by a wheeled climber, but there are potential issues in the atmosphere. This paper considers potential wind loading levels and resultant tether stresses, concluding that Earth Port retention forces are likely to exceed the strength limits of current-concept early operational tethers under foreseeable weather conditions. One calculation estimates wind loading on a 1m tether could reach 438 tonne-f, requiring an order-of-magnitude increase in tether strength and mass to maintain tether integrity. Methods to address weather-related operational issues are discussed, including the use of the Earth Port winches to raise the climber using the tether stored strain potential energy. Other options are discussed before describing one concept in more detail. The Cable Lift concept involves a pulley suspended on the lower end of the ribbon tether at some point in the lower mesosphere (45-85km altitude) creating a 'Pulley Node'. A cable is wrapped over the pulley and connected to winch systems on the Earth's surface : climber systems and payloads are then attached to the cable and raised separately to the pulley, then transferred to the tether above the pulley before final robotic assembly. The cable would be constructed using a new material with adequate specific strength, either similar to the main tether (perhaps graphene super-laminate) or an alternative more suited to atmospheric conditions (perhaps hexagonal boron nitride). The cable would be of circular cross-section with a diameter of the order of 2 mm, thus able to withstand wind loading without requiring excessive retention forces. Under calm weather conditions it would be possible to lower the Pulley Node to the surface for maintenance or upgrade : it may then be possible for climbers to ascend from the surface using either onboard motors or the stored-energy method. The Cable Lift concept should be able to operate in most weather conditions, enabling the Space Elevator to approach the target 'all weather capability'. Operations may need to be paused under extreme storm conditions, but methods are discussed that would maintain system integrity.

Keywords: Space Elevator, Atmosphere

1. INTRODUCTION

An Earth Space Elevator of length 100,000km would have less than 0.1% of its total length in the Earth's atmosphere, but that region in many ways is the most challenging part of the ascent. The tether and ascending climber could be subject to the highest effective gravity, precipitation, ice and electrical storms, and perhaps most importantly wind loading.

The wind loading issue has been recognised since the study that started the modern Space Elevator era, with Edwards and Westling proposing a tether width reduced to 20 cm in the atmosphere [1]. More recent work by Knapman [2] concluded a much narrower shape, perhaps a 4mm wire, would be required : such a small profile is needed as lateral wind loads will directly lead to an increase in the tether tension force and so require a stronger tether. This in turn will mean that the total tether mass requirement becomes far higher, potentially making the entire Space Elevator system less technically and economically viable.

Unfortunately a tether with a small profile results in substantial challenges at the tether/climber interface : a recent ISEC Study has addressed this topic in some detail, concluding that the proposed 20cm width is close to the minimum that could be climbed by a multi-tonne climber using either a friction or a non-friction drive system.

This paper quantifies the wind loading issue, summarises previous proposals, and suggests an alternative approach for ascending through the Earth's atmosphere.

2. ANALYSIS : WIND LOADING

The parts of the tether that are in the atmosphere will be subject to winds which are unpredictable and variable, but some order-of-magnitude calculations are possible.

When moving air is stopped by a surface the dynamic energy in the wind is transformed to pressure, which transforms to a force on the surface.

The pressure and force can be calculated thus [3] :

$$F_W = p_d\, A\, C_d$$

$$= 1/2\, \rho\, v^2\, A\;\; C_d \qquad \text{(Equation 1)}$$

where

F_W = *wind force (N)*

A = *surface area* (m^2)

p_d = *dynamic pressure (Pa)*

ρ = *density of air* (kg/m^3)

v = *wind speed (m/s)*

C_d = *Drag Coefficient*

The value of C_d is 1.28 for a flat plate perpendicular to a flow or '1.0-1.3' for a wire or cable [4].

Figure 1 below shows calculated wind-loading drag forces for a 0.2m wide tether from the Earth's surface to 80km altitude, using the above equation and a worse-case drag coefficient of 1.28 to calculate loads for 1 km tether elements. This coefficient is for a ribbon perpendicular to the air flow, which is clearly the worse-case condition : other orientations would generate different forces. The total of the lateral loads on all the tether elements is 247 kN.

The wind velocity profile is also shown for reference [5] : the very high wind velocities at high altitudes produce very little load on the tether as the air density is so low.

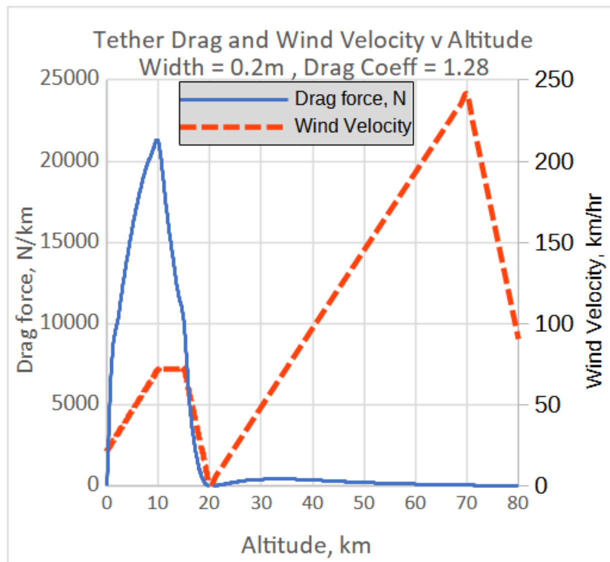

Figure 1 : Wind Pressure & Velocity .v. Altitude, 20cm (Based on unpublished analysis by L.Bartoszek)

The profile can be seen to assume a surface wind of 21.6 km/hr (6.0 m/s), Force 4 ("Moderate Breeze") on the Beaufort Scale [6]. It is uncertain if the Space Elevator system will need to routinely operate in any higher wind than this, given that equatorial winds away from land are usually very low, but any infrastructure system must be able to survive extreme events. The degree of required resilience is uncertain, given that climate change may reduce the validity of historic wind data, but tolerance to at least a 'Force 6' wind ("Strong Breeze", 10.8-13.8 m/s) is considered prudent. Thus the lateral force calculated above will be increased by a factor of (13.8/6.0)^2 = 5.29, yielding 1.31 MN.

This lateral force can be converted to a tether tensile force by considering the deflection of the tether. Knapman in 2014 [2] published the schematic reproduced in Figure 2 below, showing the deviation from the vertical produced by any lateral wind load. A target angle of 10° was proposed, and will be used here.

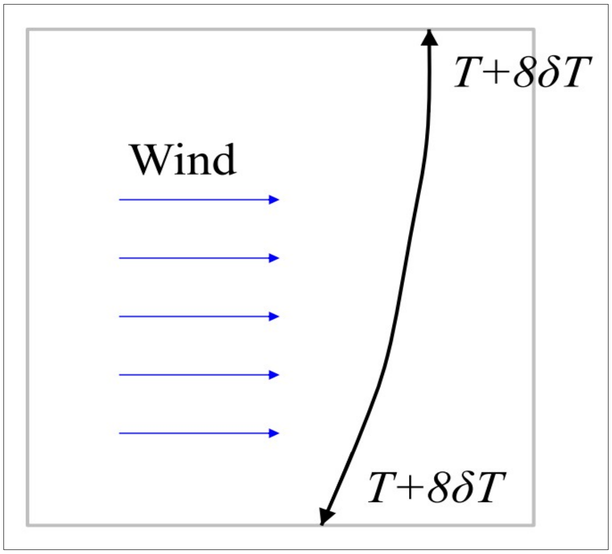

Figure 2 : Schematic of deflected tether

A simple estimate of the additional tension force can be made by assuming that the horizontal component of the additional tension force Tw, Tw * sin(10°), equals the wind load Fw. As sin(10°) = 0.1736 this yields Tw = Fw/0.1736 : hence the additional tensile force in the tether in a Force 6 wind will be 1.31 /0.1736 = 7.55 MN. At 1g this is represents a weight of 769 tonnes.

This calculation is an approximation as it assumes a horizontal wind loading force : in practice the wind load would have a vertical component as the tether is not vertical. This more precise calculation would be complex as both the tether inclination and wind velocity would vary in a non-linear manner with altitude.

The 2014 work yielded a value of 438 tonne-f extra tension, based on a different wind velocity profile. The summary conclusion is the same : a tether designed to raise a 20-tonne climber each day will typically have a working strength at the Earth Port of 35 tonne-force, and so (with a width of 20cm) will be unable to withstand sufficient wind loads.

Any increase in tether strength at the Earth Port means the tether would need to be heavier along its entire length, as it must support its own weight as well as that of any climbers. Thus the mass of the tether would need to rise from a few thousand tonnes to many tens of thousands of tonnes : this entire mass would also need to be deployed from space as wind loading would prevent a lighter 'seed' tether being deployed for construction purposes.

In summary, a light-weight climbable ribbon-type tether is not feasible for ascending through the atmosphere. Other approaches have been proposed in earlier work, summarised in the following section.

3. OTHER ATMOSPHERE SOLUTIONS

3.1 Reduced Surface Width

The previous section showed that a 0.2m ribbon-type tether in the atmosphere would be subject to excessive wind loads. The required tether cross-sectional area can be achieved in other ways to endure the full force of the wind in all conditions : adequate strength could be achieved with an effective surface width presented to the wind as little as 4 mm (perhaps achieved with a simple cable), resulting in an expected maximum wind force on the atmospheric tether of only about 26 kN. Using the 10° deflection from the vertical as before this leads to a tension increase of 151 kN, equivalent to 15.4 tonne-f.

Another option is to assume a width of 4 cm curved to present an effective width of 2 cm. Forces are then simply 10% of those described in Section 2, meaning an increase of tension of 755 kN, or 77 tonnes weight.

These two options would result in an increase in the total tether mass that might be considered acceptable, although all that mass would need to be deployed from space as wind loads would prevent the use of a 'seed' tether for construction. Unfortunately, the small surface widths of both options provide insufficient surface area for a climber to adequately grip and ascend. The 2021-2022 ISEC Study [7] has addressed this topic in some detail, concluding that the original 20cm width is close to the minimum that could be climbed by a multi-tonne climber using a friction drive system. Other non-friction drives have been assessed, but none can meet the climbability requirements.

3.2 'Spring Forward'

The 'Spring Forward' concept, first proposed by Ben Shelef in c.2009, makes use of the longitudinal elasticity of the space elevator tether and the dynamic nature of the system. The length of tether compromised by the atmosphere is less than 60km, representing less than 0.06% of the total length of a 100,000km tether. The tensioned tether could well have a nominal mean strain in excess of 6% under steady-state conditions, depending on the tether material properties and chosen working stress.

The basis of the concept is that the climber is attached to the tether at the Earth Port but does not immediately climb the tether. The Earth Port Reel-In-Reel-Out (RIRO) system is then used to reduce the tether tension below the attached climber and reel out additional tether : the climber would then rise through the atmosphere while fixed on the tether, using a small part of the tensile potential energy stored in the tether between the Earth and GEO.

The reeled-out tether below the climber does not need to be climbed and so could be of a small cross-section as described in section 3.1 above, and therefore able to withstand substantial wind loading. This means the system should be robust in all likely weather conditions, assuming the climber attachment point is above that weather.

Unfortunately the climber ascent could only take place in light wind conditions as the tether section above the climber must be of sufficient cross-sectional area to be climbed. This lack of all-weather capability means the 'Spring Forward' concept alone cannot meet the Space Elevator system operational requirements.

3.3 High Stage One ('Lofstrom Loop')

The issue of a tether in the atmosphere is avoided by two variations of the 'Multi-Stage Elevator'.

The first of these was devised by Lofstrom in his 1985 paper [8], comprising a structure built up from the Earth's surface and supported by momentum transfer from 'bolts' travelling at very high speed through the structure driven by linear motors on (or below) the Earth's surface. This structure would also support the weight of stays and other structural elements as required to withstand wind loading. Climbers would be transported up the structure to the tether attachment point at the apex outside the Earth's atmosphere.

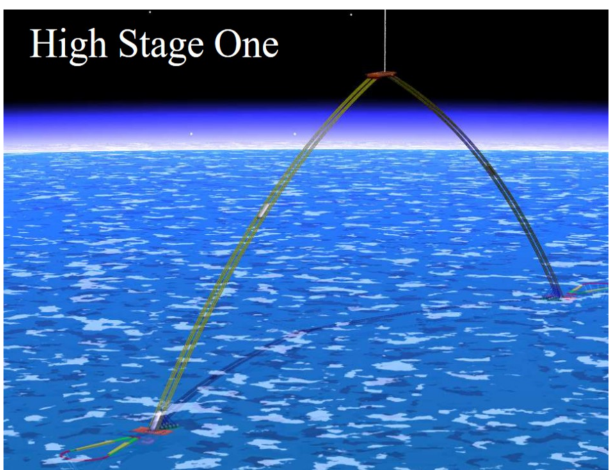

Figure 3 : High Stage One / Lofstrom Loop

The engineering of such a structure is complex and challenging, and the integrity depends on constantly moving and electromagnetically driven components. Safety, reliability and cost are major issues which make this an unlikely option in the timescale of early space elevator systems.

3.4 Multi-Stage Elevator

A variant of the Lofstrom Loop concept is the Multi-Stage Elevator system proposed by Knapman [9] [10] [11]. This also uses momentum transfer from high speed bolts accelerated by linear drivers, but in this concept the bolts travel vertically in evacuated tubes from the Earth's surface to a reversal system ('ambit') outside the atmosphere. The ambit would experience sufficient upward force to support the weight of the evacuated bolt travel tube, plus stays to combat wind loading and the weight of ascending climbers.

The concept also includes a second stage outside the atmosphere, with another reverser supported at 6000 km altitude. This second stage allows a tether to be built with less than one third of the specific strength required for the reference Earth elevator model, but has no impact on the atmospheric wind loading issue.

Figures 5 and 6 below show system schematics.

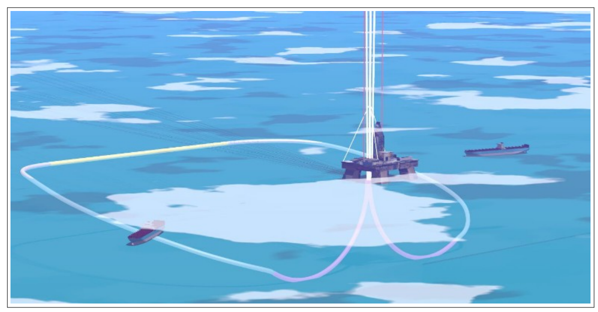

Figure 4 : Multi-stage System Base (Lower Ambit)

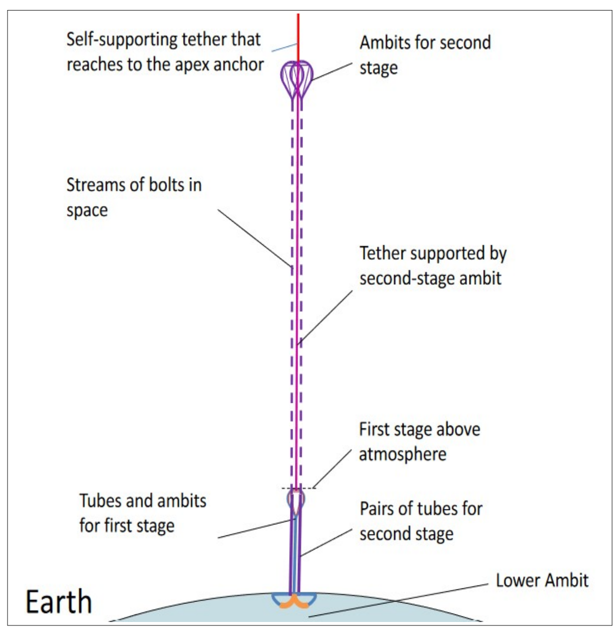

Figure 5 : Schematic of Multi-stage Elevators

This multi-stage elevator system shares a similar 'Lower Ambit' arrangement as the Loftstrom Loop, but is simpler in that it has only one ambit and does not require the 'arch' architecture. It is still very technically challenging, with high potential costs and lengthy development and construction timescales.

Early prototype work is on-going which may prove that the concept is an alternative to the high-strength material needed for a tether to the Earth's surface, but it is a complex solution to the wind loading issue.

3.5 Inflatable Towers

There have been a number of proposals and patents in recent years for inflatable structures to the edge of space. These have been put forward as means for space access, not necessarily as a high-altitude connection point for a tether to beyond GEO. Notable among these is the Thoth Tower [12], but technical details are limited with several conceptual concerns. Costs and likely development timescales make this another unlikely option for addressing the wind loading issue.

4. CABLE LIFT CONCEPT

None of the options described above fully resolve the issue of atmospheric wind loading, but Section 3.1 does conclude that a cable would withstand the wind load forces under most conditions.

4.1 Cable Lift Description

Fortunately there is a technical solution for using cables to raise multi-tonne vehicles, proven for over 80 years in terrestrial applications - the Cable Car. The proposed space elevator variant would be similar to conventional systems apart from the cable material and the vertical ascent to outside the atmosphere.

The conventional space elevator tether ribbon would terminate in the lower mesosphere (45-85km altitude) at a 'Pulley Node'. The cable would wrap around the pulley and connect to RIRO winches at the Earth Port(s), as shown in the simple schematic in Figure 6.

The cable diameter would need to be of the order of 2mm to minimise wind loading, dictating the use of a new material with high specific strength. The precise cable diameter will depend on the required system lift capacity, the material properties of the chosen material and target maximum wind loading.

The altitude of the Pulley Node is unlikely to need to exceed 60 km, and it could be raised or lowered by the RIRO winches to an optimum position based on actual and forecast atmospheric wind strengths.

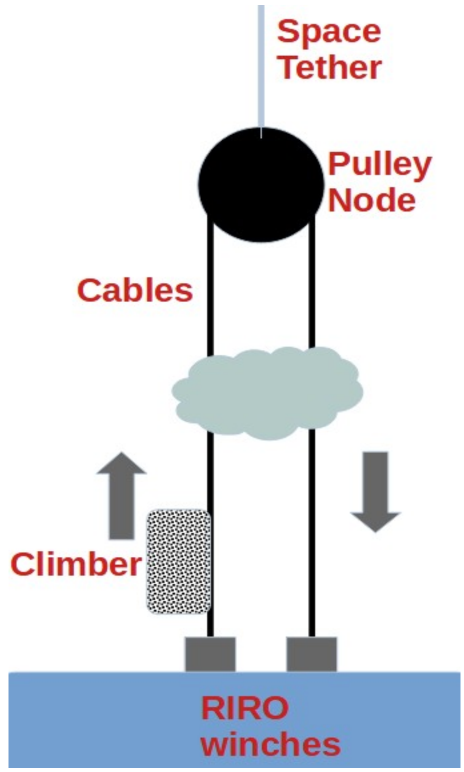

Figure 6 : Simple Schematic of Cable Lift System

The time to raise mass to the pulley should be less than two hours : one operational scenario might be to raise ten cargos of 2 tonnes each in every 24 hour period, requiring a cable speed little higher than the typical 20 km/hr of current terrestrial cable cars systems rated for human transportation.

The cable cargos would consist of either climber modules/sub-assemblies or payload, and would be transferred robotically from the cable below the pulley to the ribbon above, perhaps supported by telepresence or other control technologies. After climber attachment and payload loading the climber would commence its ascent of the ribbon above the pulley, while the cable system could immediately start raising the elements of the next climber assembly.

4.2 Cable Lift Technical Details

Precise optimisation of the Cable Lift components cannot be completed until selection of a proven ultra-strong cable material and definition of operational parameters such as the climber module masses, required mass launch rate and achievable winch speed. Table 1 below contains some technical details of an example design : hexagonal boron nitride has been chosen as the cable material due to its inert and electrical insulation properties, the foreseen higher full strength of graphene super-laminate (GSL) not being required.

Cable material	hBN
Cable diameter	2 mm
Cable density	2260 $kg.m^{-3}$
Pulley altitude	60 km
Cable mass density	7.1 kg/km
Suspended cable mass (2 x 60km)	852 kg
Earth Port cable force	10,000 kgf
Maximum Cable tensile stress	32.5 GPa
Total Pulley Cable load	20,852 kgf = 204 kN
Pulley Node Mass	500 kg
Load on base of SE tether	209 kN

Table 1 : Example Cable Lift Technical Details for '20-tonne/day' tether system

The 'Earth Port Cable Force' in the table is the maximum load on the base of each cable, comprising the tension force applied by the RIRO winch plus the weight of any mass attached to the cable. In the example above this means a mass of 2000 kg could be raised to the Pulley leaving a retention force of 8000 kgf to counter wind loading. Section 2 showed that a 20cm tether might require a tension force of the order of 500 tonne-force to withstand likely wind loading : thus, as load is proportional to width, a 2mm cable might require a retention force of 5000 kg-f, somewhat less than the surplus force in this example and so allowing scope for design optimisation or additional safety margins.

The 'Maximum Cable tensile stress' is well below the 88 GPa tether working stress proposed for GSL and so allowing the selection of hBN, although for a cable the safety margins may well need to be higher.

The 'Load on the SE tether' (comprising the tension force in both cables and the weight of the Pulley Node) can be seen to be 209kN, equivalent to a weight of just over 21 tonne. To this must be added the weight of any climber assembled on the main tether above the pulley : this is comparable to the assumed Earth Port tension in concepts where the tether extends to the Earth's surface.

4.3 Operational Options

Weather conditions are of course highly variable, and may well become more variable with future climate changes. Three possible modes of Cable Lift operations are as follows.

4.3.1 Moderate Winds – Normal Operation

Under moderate wind conditions the Cable Lift system would operate as outlined in Section 4.1 . After assembly above the Pulley Node the climber would depart and ascend to GEO. The mass of the climber and departure interval could be the full tether capacity once per day or a lower mass more frequently : see 2022 IAC Paper [13] for a full discussion of these options.

The altitude of the Pulley Node could be 60km, but with light winds it could be lowered by the Earth Port RIRO winches to accelerate the assembly process.

4.3.2 Calm Conditions – 'Spring Forward'

Under calm conditions it would be possible to lower the Pulley Node all the way to the Earth Port using the RIRO units, allowing the climber assembly and payload to be attached directly to the ribbon tether above the pulley. The pulley would then be raised back to an altitude above the altitude from which the climber could ascend as usual : this is effectively the 'Spring Forward' technique discussed in earlier studies.

While the Pulley Node was at the Earth Port there would also be an opportunity for maintenance, etc.

4.3.3 Extreme Wind Conditions – No Ascents

There will probably be occasions when wind strengths exceed normal operational limits, although Hurricane-force winds are unlikely due to the Earth Port being located on the Equator. Figure 7 below shows a NASA map of hurricane tracks up to 2006, but weather extremes associated with climate change cannot rule out high equatorial winds in the future.

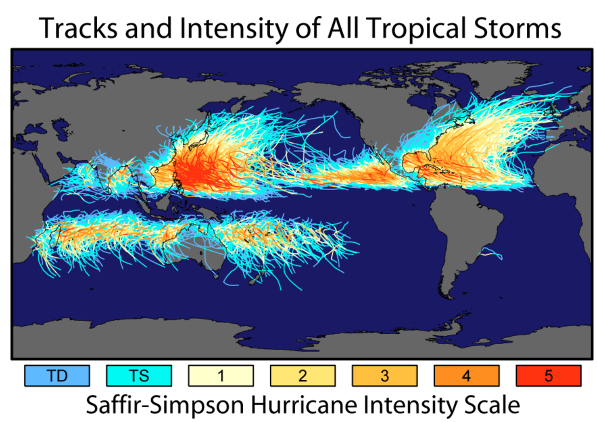

Figure 7 : Historic Hurricane Tracks to 2006
Source : NASA Earth Observatory

If excessive winds were forecast the ascent operations would be suspended first, allowing a larger cable inclination from the vertical to limit the retention force increase, but eventually even the full safe working stress of the cable could become insufficient to withstand wind loads.

One possible means of surviving stronger winds could be to replace the standard cable with one of a larger diameter. This would be beneficial as the cable strength is a function of the cross-sectional area, which of course is proportional to the square of the diameter : if wind loading is directly proportional to the cable diameter then a larger cable would withstand higher winds, albeit with substantially higher retention force. The extra weight of a heavier cable and the additional wind loading force would be offset by having no climber weight to support at the Pulley Node.

Deployment of the stronger cable would need to be planned hours in advance to allow for replacement of the standard cable. An alternative concept is described in section 4.5 below.

4.4 System Architecture Implications

One limitation of the Cable Lift concept is the mass limit for individual payload components. The numbers in Table 1 show a cable tension of 10 tonne-f, but much of this must be allocated to base tension to counter wind loading. The target payload for a 20-tonne climber is 14 tonnes, and this could not be raised as a single unit with the cable design described. Such a payload would need to await for a 'Spring Forward' launch as described in 4.3.2 : calm conditions are common on the equator, but may not coincide with operational requirements.

One possible medium-term solution to this difficulty would be to scale up the entire tether system. Figure 8 below shows a concept of six elevators, each capable of raising 20 tonnes per day.

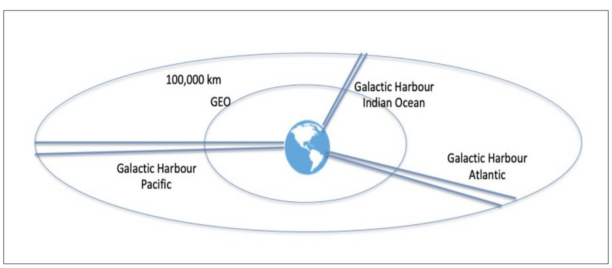

Figure 8 : Six Tether Concept (image by P.Swan)

The first of these six tether systems would be used to raise the material for the other five, but if each was rated at 20-tonnes/day the payload limitation problem of the Cable Lift concept would apply to each. This problem would be eliminated if the tether and other material for the five separate tethers were combined, perhaps into a single tether of nominal capacity 100-tonnes per day. The numbers shown in Table 1 can then be simply scaled by a factor of five, yielding Table 2 below.

Cable material	hBN
Cable diameter	4.5 mm
Cable density	2260 kg.m^{-3}
Pulley altitude	60 km
Cable mass density	35.5 kg/km
Suspended cable mass (2 x 60km)	4260 kg
Earth Port cable force	50,000 kgf
Maximum Cable tensile stress	32.5 GPa
Total Pulley Cable load	104,260 kgf = 1020 kN
Pulley Node Mass	2500 kg
Load on base of SE tether	1045 kN

Table 2 : Example Cable Lift Technical Details for '100-tonne/day' tether system

This more substantial cable system can be seen to be more than capable of raising a 14,000 kg payload as a single unit, with 36 tonne-f surface retention force in hand to counter wind loads during the ascent. The stronger cable would also mean a far higher threshold for emergency pausing of launch activities.

Note also, such a '100-tonne' tether would not necessarily be used to support single daily 100 tonne launches, at least not routinely : it may be better to rationalise on the 20-tonne climber size with five launches per day, with the added benefit of increasing the payload to orbit from 70 to 80 tonnes/day (as shown in Paper IAC-22-D4.3.68299 [12]) for the same total tether mass and improved wind tolerance.

4.5 Cable Recovery Unit

Despite the content of 4.3.3 and 4.3.4 above, increasing cable diameter may not be enough to withstand the very highest wind strengths, given that the cable weight is limited by the strength of the 'Space' tether above the Pulley Node. The Earth Port may be in an area with low likelihood of storms (see Figure 8), but climate change means that extreme weather events are becoming more frequent and less predictable. There may also be other emergencies when the Earth Port facility itself is compromised and unable to retain the tether, perhaps due to fire, earthquake or tsunami.

One emergency mitigation would be to deploy a 'Cable Recovery Unit' (CRU). This would be a mechanism with similarities to both a tether climber and the Earth Port RIRO unit, massing several tonnes and including winches connected to both ends of the cable wrapped around the Pulley Node. This CRU would detach from the Earth Port when an emergency situation is forecast and winch itself to a safe altitude. It would include power supplies, thrusters and fuel for position control while suspended from the Pulley Node. After the emergency conditions have cleared the CRU would descend back to an Earth Port for reattachment and resumption of space elevator operations.

The Recovery Unit could mass as high as 20 tonnes (for a 20-tonne/day capacity tether), given the 10 tonne capacity of each cable (see Table 1) with no additional surface retention force. For this mass it would require some on-board power supply of 1 MW to ascend at 20 km/hr. Station keeping systems and eventual descent would also need to be powered.

Such a substantial mass would also assist in keeping the entire tether system more stable.

There are many other design concepts for the CRU that could be considered in future, for example :

- the CRU could normally function as the Earth Port RIRO system, improving cost and simplicity and enabling rapid deployment in an emergency.
- the CRU could act as an escape capsule for a small number of Earth Port personnel if the impending emergency warranted rapid evacuation.
- the CRU could detach and transfer the tether attachment point from one Earth Port facility to another.

5. CONCLUSIONS

5.1 A space elevator tether system with the strength required to raise daily masses of the order of 20 tonnes will be unable to survive probable wind load forces if extended to the Earth's surface as a climbable ribbon.

5.2 Previous concept proposals for structures extending above the Earth's atmosphere are technically and economically challenging.

5.3 A 'Cable Lift' concept is proposed based on existing terrestrial cable-car concepts, extending to a 'Pulley Node' at an altitude no greater than 60 km. This will require a cable material approaching the specific strength of that required for the main space elevator tether, but otherwise will require no new technologies.

5.4 The Cable Lift system would raise elements of the space climber and cargo to the base of the 'space' tether for integration above the Pulley prior to ascent.

5.4 After construction of the first '20-tonne/day' tether, later capacity upgrades would be better achieved by constructing fewer but heavier tether systems. For the same tether mass this should provide more tolerance to extreme weather conditions and more payload lift.

5.5 The Cable concept will also enable a 'Recovery Unit' to be devised. This could detach from the Earth Port in the event of extreme weather or any threat to the integrity of the Earth Port, it would then winch itself to a safe altitude to await reconnection.

6. RECOMMENDATIONS

6.1 Further studies should review the 'Cable Lift' concept to consider the design concepts of the Pulley Node and Earth Port RIRO systems in more detail. Consideration should also be given to the optimum cable material and maximum Pulley working altitude.

6.2 The architectural concept of multiple elevator systems of equal 'Initial Operating Condition' capacity should be reviewed, with consideration given to later tethers being combined to maximise payload capability and allow more tolerance to extreme weather events.

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